AN INTEGER PROGRAMMING APPROACH FOR SINGLE TRUCK ROUTING-AND-SCHEDULING PROBLEMS TO ISLANDS WITH TIME-VARYING FERRY SCHEDULES

Mohammad Thezar Afifudin, Dian Pratiwi Sahar
1,2Department of Industrial Engineering, University of Pattimura
St. Ir. M. Putuhena, Ambon, Moluccas, 97233, Indonesia
E-mail: thezar.afifudin@fatek.unpatti.ac.id, dian.sahar@fatek.unpatti.ac.id

ABSTRACT
This study aims to develop a solving model for the single trucks routing-and-scheduling problems to islands with variations in ferry schedules. In this problem, the travel time is asymmetric and the truck routing is based on the sequence of island visits, known and unknown. The models are developed using an integer programming approach. Integer non-linear programming is formulated to solve problems where the sequence is unknown, whereas integer linear programming for the sequence is known. Besides, a delivery day scenario is built to determine the optimal route and schedule with minimum total travel time on each departure day. Numerical experiments were carried out on the case of a small distribution of a small industry in Central Moluccas, Indonesia. The results showed that the model developed could provide solutions to solve problems.

Keywords: routing; scheduling; island travel; intermoda
1. Introduction

The utilization of public transportation is often found in the distribution activities of small industries in the archipelago. To support their distribution activities to customers on different islands, they often use single-trucks and utilize existing ferry transportation. This situation is due to the scope of logistics services has not yet reached the islands. The challenge encountered was how to match the truck routes and schedules with variations of ferry schedules at each port, on each island. The aim is to minimize the delivery duration.

This problem can be defined as a single-truck assignment problem to visit markets on each different island, starting from the depot located on the first island at a certain time, and returning to the depot with minimum total travel time. Travel time between islands (by ferry line), travel time on the island (by truck line), and starting time of an island depends on the selection of ports (as of arrival and departure points) and the moment of times (departure time and sequence). All three affect the length of visit or delay time on an island. From this definition, this problem can be classified as a variant of the time-dependent traveling salesman problem (see Malandraki & Daskin, 1992).

Every island to be visited has at least one ferry port. Of all the available ferry ports, some can only be connected by land and some by sea. Each ferry port has a varied schedule based on the port of destination, day of operation, and hour of departure. The consequences of delays cause trucks to choose another alternative port or wait for the next trip. Meanwhile, to start the overland journey from the arrival point, the truck is not constrained by the waiting time.

Literature related to the TDTSP is still rarely found. Since it was introduced by Malandraki & Daskin (1992) using MILP and some simple heuristics, up to now, TDTSP has been developed on the approach or method used (e.g. Harwood et al., 2013; Taş et al., 2016), with time windows (e.g. Albiach et al., 2008; Cordeau et al., 2014; Montero et al., 2017; Ban, 2019), and its generalizations. Generalization of TDTSP was carried out by researchers to solve single-machine scheduling or production problems (e.g. Gouveia & Voß, 1995; Bigras et al., 2008; Stecco et al. 2008; Stecco et al., 2009; Miranda-Brønt et al., 2010; Abeledo et al., 2013; Godinho et al., 2014), moving targets TSP (e.g. Helvig et al., 2003; Montemanni et al., 2007; Hassoun et al., 2019), time-dependent orienteering problem (e.g. Verbeeck et al., 2014; Verbeeck et al., 2017), and controlled airspace (e.g. Furini et al., 2016). A recent survey of TDTSP variants can be seen in Gendreau et al. (2015).

From the literature above, TDTSP is depicted as a graph consisting of a set of vertices and a set of directed links, where nodes are not restricted by region. In this study, the problem is described as a graph consisting of a set of regions (islands), a set of nodes (ports), and a set of directed links connecting regions and nodes. Each region has at least one node to be chosen as a connecting point (arrival and departure) with other regions.

Another characteristic of this problem is sequence-dependent, where the starting time of an island and the time of its visit depend on the completion of the visit on the previous island. This characteristic was considered by Picard & Queyranne (1978) to solve tardiness in single-machine scheduling problems. In this problem, the travel time (setup time) between two nodes depends not only on the two respective locations involved but also on their position in the sequence. Exact approaches for variants of this problem can be found in Gouveia & Voß (1995), Abeledo et al. (2013), Miranda-Brønt et al. (2010), and Godinho et al. (2014). However, according to Picard & Queyranne (1978), the single-machine sequence problem is TDTSP. This is different from the characteristics and properties stated by Malandraki & Daskin (1992), where the travel time between two nodes depends on the moment of the day in which the arc is traversed.

Stecco et al. (2008) distinguished more contrasts regarding sequence-dependent characteristics in single-machine scheduling problems. In the cases they studied, the transition between two jobs involved two types of arrangements, finite and indefinite. The sequence-dependent characteristic is seen when setting up two jobs. In scheduling work, it is important to know the predecessor turnaround time, which determines whether the two arrangements can be performed sequentially without interruption or there is a wait time due...
to setting constraints. They formulate a model using a branch-and-cut algorithm to minimize the turnaround time of the last job. Meanwhile, Stecco et al. (2009) formulated a problem with the same characteristics and objectives using the tabu-search heuristic.

In this paper, we generalized the TDTSP by considering sequence-dependent. We use the integer programming approach to formulate this problem. The formulation was developed for two conditions, the sequence of visiting the island is known and not. We developed the INLP model for conditions where the sequence of visits is known and the ILP model where the sequence is unknown. Numerical experiments are performed on a small case that is often faced by the small industries in the Moluccas islands, Indonesia.

The contributions of this paper are: 1) introducing new TDTSP variants by considering the sequence of visits, time-varying schedules, and regional constraints and 2) developing a problem model using integer programming approaches for conditions where the sequence of visits to islands is known and unknown.

For a more detailed discussion, we divide the contents of this paper into the following sections. In Section 2, we describe the problem in detail and explains the technique and model formulation. Section 3 discusses the results of a numerical experiment on a case. Finally, in section 4, we present conclusions and future research.

2. Research Methods
The focus of this research is to develop a model to solve the routing and scheduling problems of single-trucks to islands using ferries. This research is divided into three important stages, namely: data collection and processing, model development, and model validation.

2.1. Data Collection
Data collection and processing were carried out through observation, interviews, and literature review related to the problem of single-truck routing and scheduling to the islands.

Observations were made on cases that are often faced by small industries in archipelagic areas. Figure 1 illustrates the characteristics of the problem. To resolve this case, we conducted interviews with industry players and reviewed related literature. The results show that the problems faced are a new variant of the TDTSP. Of the delivery cases obtained, concerning the order of travel, some were determined by the company (known sequence) and some were not (unknown sequence).

![Figure 1. Characteristics of the problem](https://via.placeholder.com/150)

For the purposes of developing the model, we refer to the formulation principles proposed by Malandraki & Daskin (1992) related to TDTSP and Stecco et al. (2008) related to sequence-dependence.

2.2. Model Development
Model development starts from identifying variables, compiling notations and indexes, explaining the techniques used in formulations, formulating problems in mathematical form, to developing formulas through integer programming. We developed the INLP model for conditions where the visit sequence was known and the ILP model where the sequence was unknown. Model programming is assisted by standard office computers and Lingo software.

2.3. Model Validation
Model validation is done through numerical model testing to see the suitability between input parameters, decision variables, and expected goals. A numerical test was carried out on one of the shipping cases facing a small industry. The data required includes ferry schedules and travel times between ports. Ferry schedules and sea route travel times were obtained from telephone interviews with PT ASDP Ambon Branch, while inter-port travel times for land routes were obtained using https://www.google.com/maps.
3. Results and Discussion

3.1. Notations dan Formulation

The problem can be described as follows. Let $A$ be a set of sequences and $B$ be a set of islands, where the size of the sequences and the islands is $L, A = B = \{1,2, ..., L\}$. Each sequence is indexed with $e \in A$, while each island is indexed with $f$ or $g \in B$ (see Fig. 3). Let $C$ be a set of ferry ports of size $M, C = \{1,2, ..., M\}$. Each port is indexed with $b, i, j \in C$. Let $D$ be the set of ferry departure times of size $N, D = \{1,2, ..., N\}$. Each departure time is indexed with $k (k \in D)$. $D$ is a projection of all ferry schedules (including hours and days) at each port on the time horizon.

The parameters, decision variables, and additional variables used in the formulation are denoted as follows:

Parameters:

\[ s_{hi} = \begin{cases} 1, & \text{if port } b \text{ and port } i \text{ is connected} \\ 0, & \text{otherwise} \end{cases} \]

\[ t_{hi} = \text{travel time from port } b \text{ to port } i \text{ (including sea and land)} \]

\[ u_{ijf} = \begin{cases} 1, & \text{if port } b \text{ is located on island } f \\ 0, & \text{otherwise} \end{cases} \]

\[ v_{ijk} = \begin{cases} 1, & \text{if travelling from port } b \text{ to port } i \text{ can be done at the ferry departure time } k \\ 0, & \text{otherwise} \end{cases} \]

\[ w_{k} = \text{ferry departure time } k \]

\[ R = \text{the last departure time on the first day on the first island.} \]

Decision variables:

\[ x_{ki} = \begin{cases} 1, & \text{if the route from port } b \text{ to port } i \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \]

\[ y_{ijk} = \begin{cases} 1, & \text{to island } g \text{ using schedule } k \\ 0, & \text{otherwise} \end{cases} \]

\[ \zeta_{e} = \begin{cases} 1, & \text{if island } f \text{ is in the sequence } e \\ 0, & \text{otherwise} \end{cases} \]

Auxiliary variables:

\[ ST_{e} = \text{starting (departure) time from island in the sequence } e \]

\[ TT_{f} = \text{time to visit island } f \text{ (including sea and land) from the previous island.} \]

\[ t'_{i} = \text{travel time from depot to port } i \]

Time-dependent characteristics can be seen in inter-island travel time (by ferry line) and travel time on the island (by truck line). Both depend on ports selection (as arrival and departure points) and moment of times (departure and order times). Travel time by ferry from island $f$ to island $g$ and travel time by truck on island $g$ can be determined based on conditions if the sequence of island $g$ is after island $f (\zeta_{e} = \zeta_{e-1} = 1)$, travel time $t_{e}$ can be determined based on conditions if the sequence of island $g$ is after island $f (\zeta_{e} = \zeta_{e-1} = 1), travel time line}$ from island $f$ to island $g$ on schedule $k$ is selected ($y_{ijk} = 1$), departure time $k (w_{k})$ is available and can connect port $b$ to port $i (y_{ijk} = 1)$, port $b$ is located on island $f$ and port $i$ and port $j$ are located on island $g (w_{j} = w_{k} = w_{k} = 1)$, port $b$ to port $i$ and port $i$ to port $j$ are connected and selected ($s_{bi} = s_{ij} = x_{bi} = x_{ij} = 1$). Therefore, the travel time from island $f$ to island $g$ selected is $t_{b}$ while the travel time on island $g$ selected is $t_{g}$.

The sequence-dependent characteristics can be seen at the starting time of each island. The sequence-dependent starting time of the island in the sequence $e$ occurs in the schedule if the departure time of the island is processed right after the end of the island's visit from the island in sequence $e-1$. As with travel time, the starting time ($ST$) also depends on the selection of ports and moment of times. Starting time is the selected departure time $k$. If referring to the conditions above, where $\zeta_{e} = \zeta_{e-1} = 1$, $t_{b}$ and $t_{g}$ have been determined, then the starting time of the island $g$ in sequence $e$ ($ST_{e}$) must meet the inequality $ST_{e} > ST_{e-1} + t_{b} + t_{g}$. This inequality explains that the three time-variables affect the length of time to visit island $g (TT_{e})$ or delay time before departure from island $g$.

The objective function of minimizing total travel time is built based on the starting time of the last and first sequence islands and the length of visit time on the first sequence island. It is assumed that the starting time of the depot depends on the starting time on the first island. Thus, the delay time on the first island is zero. If the number of islands or sequences is equal to $L$, then the total travel time is $ST_{L} - ST_{1} + TT_{1}$. (see Fig. 3).

The start time of the depot and the time of arrival at the depot can be determined if the distance from the depot to the depature port on the first island is known. If the travel time from the depot to the departure port is $t'$, the start time from the depot ($OS$) is $ST_{1} - t'$ and the time to arrive at the depot ($OF$) is $ST_{L} + TT_{1} - t'$. 56
3.2. Mathematical Formulation

The model was developed for two conditions, the sequence of islands is known and unknown. For unknown sequences, the model was developed with INLP, while for known sequences, the model was developed with ILP.

3.2.1. The INLP model for unknown sequence problems

The INLP model is shown below:

Objective:

$$\text{Minimize} \quad \left( \sum_{e \in A} \sum_{e \in A} S_T^e - \sum_{e \in A} S_T^e \right) + \sum_{f \in B \setminus \{1\}} \sum_{f \in B \setminus \{1\}} T_T f$$ \quad (1)$$

subject to,

$$\sum_{h \in C \setminus C_{e f}} x_{h \in C} u_{h f} u_f = 1; \forall f \in B \quad (2)$$

$$\sum_{f \in B \setminus C} x_{h \in C} u_{h f} u_f = L \quad (3)$$

$$\sum_{f \in B \setminus C} x_{h \in C} u_{h f} u_f = 1; \forall f \in B \quad (4)$$

$$\sum_{f \in B \setminus C} \sum_{g \in B \setminus f} x_{h \in C} u_{h f} u_f = \sum_{f \in B \setminus C} \sum_{g \in B \setminus f} x_{h \in C} u_{h f} u_f \quad (5)$$

$$\forall f \in C \quad (6)$$

$$\sum_{f \in B \setminus C} \sum_{g \in B \setminus f} x_{h \in C} u_{h f} u_f = \sum_{i \in C} \sum_{j \in f} x_{h \in C} u_{h f} u_f \quad (8)$$

$$\forall f \in C \quad (7)$$

$$\sum_{k \in D} y_{f k} \leq 1; \forall f, g \in B \quad (9)$$

$$\sum_{k \in D} y_{f k} = 1; \forall f \in B \quad (10)$$

$$\sum_{j \in f} y_{f k} = L \quad (11)$$

$$\sum_{j \in f} y_{f k} \leq \sum_{i \in C} \sum_{j \in f} x_{h \in C} u_{h f} u_f v_{h k} \quad (12)$$

$$\forall f, g \in B, f \neq g, \forall k \in D \quad (13)$$

The objective function (1) minimizes the total travelling time. Constrains (2-3) show that land trips on each island, from port to port, can only be done once and the total is following the number of islands. Constraints (4-6) show that sea travel between different ports of the island is only done once and ensures that each port on each island will only be a point of departure to the next island if visited before from the port of the island with it and vice versa. Constrains (7-10) show that each island only chooses one departure time and ensures that the departure time is not less than the cumulative travel time to the island’s departure port. Constrains (11-13) ensure that each island has only one sequence number and the origin island is first. Constrains (14) is the total travel time to the next island (sea and land), starting from the time of departure from the previous island to the time of departure from the island. Constrains (15-17) ensure that the departure time on an island is not more than the departure time on the island in the previous sequence. Constrains (18-20) show that the three decision variables ($x_{h \in C}$, $y_{f k}$, $z_{f k}$) are binary.
Functions (1) to (20) are general formulations for finding the minimum total travel time. Meanwhile, functions (21) and (22) are additional functions to determine the start time from the depot (OS) and the arrival time at the depot (OF).

3.2.2. The ILP model for known sequence problems

The ILP model for a known sequence is developed from the INLP model by making $z$ as an input variable and removing constraints (11-13) and (18).

3.3. Computational Experiments

Computational experiments were performed on the case of product distribution of one of the small industries in central Moluccas, Indonesia. Products will be distributed to two markets (Ambon City and Masiho City) which are located on two islands (Ambon Island and Seram Island) different from the island where the industry is located (Saparua Island). To support the distribution of their products, the company uses a single truck and utilizes ferry transportation services. There is only one port on Saparua Island (Umeputih), there are two on Ambon Island (Amahai and Hunimua), while on Seram Island there are four (Amahai, Ina-marina, Wailey, Waipirit) which are connected close to Saparua and Ambon islands.

Each port has a varied ferry shipping schedule and some of them are interconnected (see Table 1). Travel time from the arrival port to (the market/customers and then return to) the destination port is shown in Table 2. In this case, the ferry schedules are fixed every week. We built seven scenarios based on the day of departure to find out the right route-and-schedule decisions on each day of departure. We use a time horizon of 3 days based on the estimation that the visit time for each island is at most one day. The travel time for two unconnected ports is given as 20 hours (Q or big value).

Table 1. Ferry schedules

<table>
<thead>
<tr>
<th>Islands</th>
<th>Ports</th>
<th>Destination Ports</th>
<th>Departure times</th>
<th>Days of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saparua</td>
<td>Umeputih</td>
<td>Amahai</td>
<td>12:00</td>
<td>Monday, Tuesday, Thursday, Friday, and Saturday</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wailey</td>
<td>11:00, 13:00</td>
<td>Everyday</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waa1</td>
<td>15:00</td>
<td>Monday, Tuesday, Thursday, Friday, and Saturday</td>
</tr>
<tr>
<td>Seram</td>
<td>Amahai</td>
<td>Umeputih</td>
<td>09:00</td>
<td>Wednesday and Sunday</td>
</tr>
<tr>
<td></td>
<td>Ina-marina</td>
<td>Hunimua</td>
<td>14:00</td>
<td>Everyday</td>
</tr>
<tr>
<td></td>
<td>Wailey</td>
<td>Umeputih</td>
<td>10:00, 12:00</td>
<td>Everyday</td>
</tr>
<tr>
<td></td>
<td>Waipirit</td>
<td>Hunimua</td>
<td>05:30, 08:00, 10:30, 13:00, 15:30, 18:30, 21:00</td>
<td>Everyday</td>
</tr>
<tr>
<td>Ambon</td>
<td>Waai</td>
<td>Umeputih</td>
<td>09:00</td>
<td>Monday, Tuesday, Thursday, Friday, and Saturday</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hunimua</td>
<td>Ina-marina</td>
<td>Waipirit</td>
<td>08:00</td>
<td>Everyday</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>05:00, 08:00, 10:00, 13:00, 15:30, 18:30, 21:00</td>
<td>Everyday</td>
</tr>
</tbody>
</table>

Table 2. Travel time between two port (in hour)

<table>
<thead>
<tr>
<th>$t_{h}$ (hour)</th>
<th>Umeputih</th>
<th>Amahai</th>
<th>Ina-marina</th>
<th>Wailey</th>
<th>Waipirit</th>
<th>Waai</th>
<th>Hunimua</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umeputih</td>
<td>1.6</td>
<td>6</td>
<td>0.89</td>
<td>0.93</td>
<td>3.51</td>
<td>4.79</td>
<td>Q</td>
</tr>
<tr>
<td>Amahai</td>
<td>6</td>
<td>0.89</td>
<td>0.93</td>
<td>3.51</td>
<td>4.79</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td>Ina-marina</td>
<td>Q</td>
<td>0.68</td>
<td>0.52</td>
<td>3.33</td>
<td>4.61</td>
<td>Q</td>
<td>4</td>
</tr>
<tr>
<td>Wailey</td>
<td>1</td>
<td>3.45</td>
<td>3.5</td>
<td>6.44</td>
<td>7.72</td>
<td>Q</td>
<td>Q</td>
</tr>
<tr>
<td>Waipirit</td>
<td>Q</td>
<td>4.75</td>
<td>4.78</td>
<td>7.73</td>
<td>9</td>
<td>Q</td>
<td>1.5</td>
</tr>
<tr>
<td>Waal</td>
<td>Q</td>
<td>4</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
<td>2.9</td>
<td>2.95</td>
</tr>
<tr>
<td>Hunimua</td>
<td>Q</td>
<td>4</td>
<td>Q</td>
<td>Q</td>
<td>1.5</td>
<td>2.94</td>
<td>2.99</td>
</tr>
</tbody>
</table>

| Q              | No linked, big value Q | By truck | By ferry |

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This problem was solved using the INLP model because the sequence of visits to the island were unknown. The results show that the optimal total delivery time starting on Monday is 21.1 hours. The same results are shown if the delivery starts on Tuesday, Thursday, Friday, and Saturday. For deliveries on a Wednesday or Sunday, the optimal total time is increased to 25.1 hours.

The route and scheduling of trucks can be seen in Figure 3. The route of truck trips on the first-day group (Monday, Tuesday, Thursday, Friday, and Saturday) places Ambon island in second place and Seram island in third, while on the second-day group (Wednesday and Sunday) places the two islands in a different order. The results on the truck schedule show that the sea and land travel times for an island and the starting time from an island (ST) depend on the moment of day and sequence. The three time variables affect the length of visit (TT) and waiting time on an island.

The match between the value of the objective, route, and schedule with the given parameter values indicates that the developed model is valid through numerical testing. Whereas the difference in the value of the two groups of days, on sea and land travel time for an island and the starting time from an island (ST) proves that this problem is a development of the time-dependent traveling salesman problem (TDTSP) introduced by Malandraki & Daskin (1992), and the sequence-dependent time-dependent traveling salesman problem (STDTPS) introduced by Stecco et al. (2008).

For the ILP model, the same results are obtained if we assume that the sequence for each island is known based on the INLP results. In the computer and the same case, the INLP model requires 252 Kb of generator memory to solve the unknown sequence problem for 21 seconds, whereas the ILP model requires only 244 Kb of generator memory to solve the unknown sequence problem for 1 second.

Figure 3. Optimal solution

4. Conclusions

The problem of route and scheduling of single trucks to islands using ferry transportation is a generalization of the TDTSP by considering sequence-dependencies and regional constraints. The experimental results show that the sea and land travel time of an island and the starting time from an island (ST) depend on the moment of day and sequence. The three time variables affect the length of visit on an island and the waiting time.

The INLP model developed can be used to solve problems where the sequence of island visits is unknown. Meanwhile, the ILP model can be used for problems where the sequence of island visits is known or predetermined. Especially for shipping problems in island areas, this model is very useful for optimizing the total delivery time.

For future development, several constraints that can be considered including container or truckload capacity, commodities or product types, integration with trips to customers in each market or city, maintenance
effects on ferry operations, and how to synchronize with other modes of transportation. Also, other solution methods or approaches are needed to solve problems on a large scale.

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